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The pressure drop created by a ball settling in a quiescent suspension of comparably sized spheres

By L. A. MONDY,[1] N. TETLOW,[2] A. L. GRAHAM,[2] J. ABBOTT[2] AND H. BRENNER[3]

[1] Sandia National Laboratories, Albuquerque, NM 87185-0834, USA

[2] Los Alamos National Laboratory, Los Alamos, NM 87145, USA

[3] Department of Chemical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139-4307, USA

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Measurements are reported of the pressure differences ΔP existing at large distances above and below a ball settling along the axis of a circular cylinder filled with an otherwise quiescent viscous Newtonian liquid in which identical particles, comparable in size to the settling ball, are suspended. The suspensions ranged in solids volume fraction ϕ from 0.30 to 0.50 and consisted of 0.635 cm diameter spheres density-matched to the suspending oil. The settling balls varied in diameter from 0.318 to 1.27 cm, resulting in particle Reynolds numbers always less than about 0.4 based upon ball diameter and the effective viscosity of the suspension. For the moderately concentrated suspension ($\phi = 0.30$), the product of ΔP with the cross-sectional area A of the containing cylinder was observed to be equal to twice the drag force D on the settling sphere, in accord with theory. In the more concentrated suspension ($\phi = 0.50$) this product was found to be slightly, but significantly, less than twice the drag on the settling sphere. It is speculated that this lower pressure drop may result from the presence of one or more of the following phenomena: (i) migration of the falling ball off the cylinder axis; (ii) apparent slip of the suspension at the cylinder wall; (iii) blunting of the otherwise Poiseuillian parabolic velocity profile, the latter phenomenon being known to occur during the creeping flow of concentrated suspensions through circular tubes. Incidental to the suspension experiments, for a homogeneous fluid we verify the classical theoretical formula for the off-axis pressure drop when the sphere settles at a non-concentric position in the cylinder.

1. Introduction

The dynamic pressure difference ΔP existing between any two planes situated at large distances above and below a spherical particle settling slowly along the axis of a long circular tube filled with a homogeneous static viscous liquid has been the subject of several theoretical and experimental investigations. In this study we compare these 'single-phase' pressure drop results with our measurements for suspensions of neutrally buoyant spheres.

The Stokes (Brenner & Happel 1958; Brenner 1959) and Oseen (Brenner 1962) equations predict that

$$\Delta P A = 2D, \tag{1}$$

for a relatively small particle settling along the axis of a circular tube of radius R_0, where $A = \pi R_0^2$ is the cross-sectional area of the cylinder ($R_0 = 7.16$ cm in the present study). This theoretical prediction has been verified experimentally for rigid spherical particles falling along the axes of cylinders filled with Newtonian liquids; the relation holds for Reynolds number $Re < 125$ (Pliskin & Brenner 1963; Feldman & Brenner 1968), where $Re = 2aU\rho/\mu$ (a is the falling ball radius, U the terminal settling velocity, ρ the fluid density, and μ the viscosity). When a/R_0 is not small compared with unity, the ball experiences a wall effect, and (1) (in creeping flow) requires modification such that the pressure drop/drag force coefficient

$$C_P \equiv \Delta PA/D \tag{2}$$

takes the form (Pliskin & Brenner 1963)

$$C_P = 2\left[1 - \frac{2}{3}\left(\frac{a}{R_0}\right)^2\right] + O\left(\frac{a}{R_0}\right)^3. \tag{3}$$

Again for creeping flow, Brenner & Happel (1958), Brenner (1962), and Bungay & Brenner (1973) theoretically investigated the effects of the tube wall for non-axial settling, where the ball is positioned eccentrically in the tube. In this case

$$C_P = 2\left[1 - \left(\frac{b}{R_0}\right)^2 - \frac{2}{3}\left(\frac{a}{R_0}\right)^2\right] + O\left(\frac{a}{R_0}\right)^3, \tag{4}$$

where b is the distance of the sphere centre from the cylinder axis.

The corresponding axial fall pressure differences occurring for (single-phase) non-Newtonian fluids were studied numerically by Zheng, Phan-Thien & Tanner (1991) and experimentally by Ribeiro, Vargas & Frota (1994). The numerical results yielded $C_P = 2.016$ after correcting for wall effects (equation (2)) for the Newtonian Stokes flow case, and predicted a slight increase with the onset of both inertial and non-Newtonian effects, as represented by the Reynolds and Weissenberg (Wi) numbers. However, at zero Re the increase was not great over the range of parameters studied; C_P increased less than 10 % as Wi increased from 0 to 1. For an inelastic shear-thinning fluid (a Carreau model with a power-law index of 0.38 and a time constant that varied from 0 to 10) at zero Re, they also predicted C_P to be somewhat higher than 2. Ribeiro *et al.* (1994) argued that the effect of shear thinning should be to decrease C_P (significantly for a power-law index of 0.38, the value used by Zheng *et al.* (1991)), basing this argument on the analysis by Brenner (1962) together with the expected velocity profile for axial flow between concentric cylinders. Ribeiro *et al.* (1994) suggested that the inner cylinder could be taken as representing the falling ball. The experiments of the latter group performed with an elastic shear-thinning fluid, namely a solution of a polyacrylic acid in glycerol with a power-law index ($1/s$) of 0.874, resulted in a measured C_P of 1.95 ± 0.05. This was close to their theoretically predicted value of $C_P = (s+3)/(s+1) = 1.93$ for a fluid with a power-law index of 0.874, as well as to the Newtonian value for C_P (even if uncorrected for the effects of the non-zero a/R_0). With viscoelastic and shear-thinning solutions corresponding to two concentrations of polyacrylamide in water, the experimental results showed C_P to be about 2.0 at lower values of Re and Wi, increasing to approximately 2.3 at the highest values studied. This C_P ratio was much higher than they predicted theoretically. However, a third solution, entailing the highest concentration of polyacrylamide employed (10^4 p.p.m. by weight), yielded a significantly lower value of C_P (near 1.5 at the same conditions that yielded $C_P \approx 2$ for the other solutions), a value close to their predicted value for small Re and Wi.

It is interesting to note that the discrepancy between (1) and the comparable relation $\Delta PA = D$ (that would obtain for a laterally 'unbounded' fluid or in a cylinder with perfect 'slip' at the walls) derives from the existence of a finite shearing force on the vertical walls of the cylinder, even when these walls are effectively infinitely far from the particle, i.e. $a/R_0 \to 0$ (Brenner 1962). This shearing force arises from a reverse flow near the walls, compensating for the fluid dragged along by the particle. It is also interesting to note that when the particle has reached its terminal settling velocity in a Newtonian liquid, ΔP does not depend on the fluid viscosity, but only on the drag force $D = mg$, where m is the mass of the particle corrected for buoyancy and g is the acceleration due to gravity. This fact, coupled with the relatively weak dependence of ΔP on any non-Newtonian behaviour, suggests that the C_P ratio measured for a ball falling through a suspension may be insensitive to the volume fraction of suspended solids (which changes the suspension's apparent viscosity), provided that the presence of the suspended particles does not create any apparent suspension-scale slip at the tube wall.

Such a weak effect would be consistent with the findings of Mondy, Ingber & Dingman (1991), who calculated numerically the pressure drop across spheres falling in relatively dilute ($\phi \leqslant 0.05$) suspensions of similarly sized spheres dispersed in a Newtonian liquid. In these simulations, the pressure drop was influenced only weakly by the presence of these neutrally buoyant particles. The arrangement of the suspended particles also had a negligible effect on the pressure drop. In contrast to ΔP, however, the apparent viscosity of the suspension (as measured by the terminal velocity of the same falling balls) was observed to be very sensitive to both particle arrangement and concentration.

The present work constitutes an attempt to confirm the applicability of (1) for the case of a ball falling slowly through a suspension of neutrally buoyant spheres in a viscous Newtonian liquid. The suspended particles (radii $a_s = 0.318$ cm) were similar in size to the falling balls (radii a ranging from 0.159 to 0.635 cm). Under similar circumstances, Mondy, Graham & Jensen (1986) measured the terminal velocities of the falling balls and, hence, via Stokes law, the suspension's apparent viscosity. They observed the (distance/time) averaged velocity of a ball to be consistent with that for a hypothetical homogeneous Newtonian liquid, including theoretical wall effect corrections over the range of a/R_0 values encountered in the present study. However, they also observed that the ball's instantaneous velocity could vary dramatically as the ball interacted locally with the suspended particles in its neighbourhood, the velocity depending *inter alia* on how close the ball was to a suspended particle or group of particles at that instant. Abbott (1993) later investigated the fluctuations in ball velocity in more detail, quantifying them statistically by a dimensionless pair of Taylor dispersivities (longitudinal and transverse to the cylinder axis) dependent upon a/a_s and ϕ. In general, these dispersivities increased with increasing ϕ and decreased with increasing a/a_s. This existence of a transverse component of the dispersivity dyadic implies that the ball can drift off the centre of the cylinder axis, potentially altering its instantaneous b/R_0 value appearing in (4) during its fall. Our goal was to investigate whether or not the discrete nature of the suspensions influenced the pressure drop coefficient C_P, in comparison with a truly homogeneous fluid continuum characterized by comparable physical properties.

In an interesting study, Poletto & Joseph (1995) have investigated the motion of balls falling through suspensions of non-neutrally buoyant particles undergoing sedimentation or fluidization. The emphasis in their study was on the effective viscosity and effective buoyancy forces encountered by a ball during its motion through the sedimenting or fluidized suspension. In contrast, our work is concerned only with neutrally buoyant suspensions. Moreover, the effective viscosity does not enter explicitly into our study, nor does the question of suspension-scale buoyancy force arise, except insofar as the fact that steady-state drag D on the ball is necessarily equal to the weight of the ball corrected for buoyancy. However, because the suspended particles and fluid have the same density in our experiments, there can be no issue as to what is the proper suspension-scale buoyancy force to use in calculating D.

The next section describes the equipment and experimental protocols. In the following section we present results, including an analysis of the possible influence of the falling ball's transverse trajectory on the wall effects. The final section offers a summary and brief discussion of the results.

2. Equipment and experimental procedure

Figure 1 provides a sketch of the experimental apparatus. It consisted of five main parts: (i) glass column with an inside diameter of 14.32 cm ($R_0 = 7.16$ cm) connected near its base to a 3.81 cm diameter side column; (ii) a pressure transducer between the two columns; (iii) a temperature control system; (iv) an electromagnetic ball release at the top of the larger column; and (v) a data acquisition system.

The larger column was filled with the test liquid, either the suspending Newtonian oil alone or a suspension with a solids volume fraction of 0.30 or 0.50. The suspending fluid consisted of a mixture of 50.27% by weight of Triton X-100 (an alkylaryl polyether alcohol from J. T. Baker), 35.66% by weight of UCON oil (H-90000, a polyalkylene glycol made by Union Carbide), and 14.07% by weight of a solution of practical grade 1,1,2,2 tetrabromoethane (from Eastman Kodak) together with a small amount (about 0.1% of the weight of the tetrabromoethane) of Tinuvin 328 (an antioxidant made by Ciba-Giegy). This composition was chosen to match the density of the polymethyl methacrylate suspended spheres at the operating temperature (17.90 °C). It also has the advantage of being transparent because the index of refraction of the liquid is close to that of the suspended particles. The suspending liquid viscosity at this temperature is 6.5 Pa s, and has been shown previously (Abbott *et al.* 1991) to exhibit no shear-rate dependence or normal stresses at the shear rates encountered in the experiments. The suspended particles consisted of individually ground, uniform spheres of radius 0.318 cm obtained from Clifton Plastics (Clifton Heights, PA). Prior to dropping each ball the suspensions were well mixed (by hand) in the large cylinder to ensure that the suspended particles were uniformly dispersed.

The smaller column was filled with only the suspending Newtonian oil, and the two glass columns were connected near the bottom via a glass neck containing a small screen that prevented suspended particles from migrating between columns. This prevented any uncertainty in the initial volume fraction of solids in the test suspension (it was not possible to adequately mix the suspension in the small diameter cylinder). Balls were dropped in the larger cylinder, with the smaller column serving to balance the hydrostatic pressure, thereby enabling the minuscule differential pressures existing across the falling ball to be measured accurately. Above the liquid level in each cylinder was an air space, about 5 cm in height. Each column was capped with an airtight seal; a tube connected the air space of each cylinder to the two pressure ports of the differential pressure transducer, as well as to a bypass valve open to the atmosphere.

During an experiment the bypass valve was closed, and the two cylinders sealed off from the atmosphere. An MKS 398 differential high-accuracy pressure transducer (Andover, MA) was used to measure the pressure difference between the respective air spaces in the two cylinders. The manufacturer claimed this gauge to have a maximum error of $\pm 0.05\%$ of the transducer reading down to extremely low pressures. However, the instrument was only factory calibrated down to 1.333 Pa. Because we expected

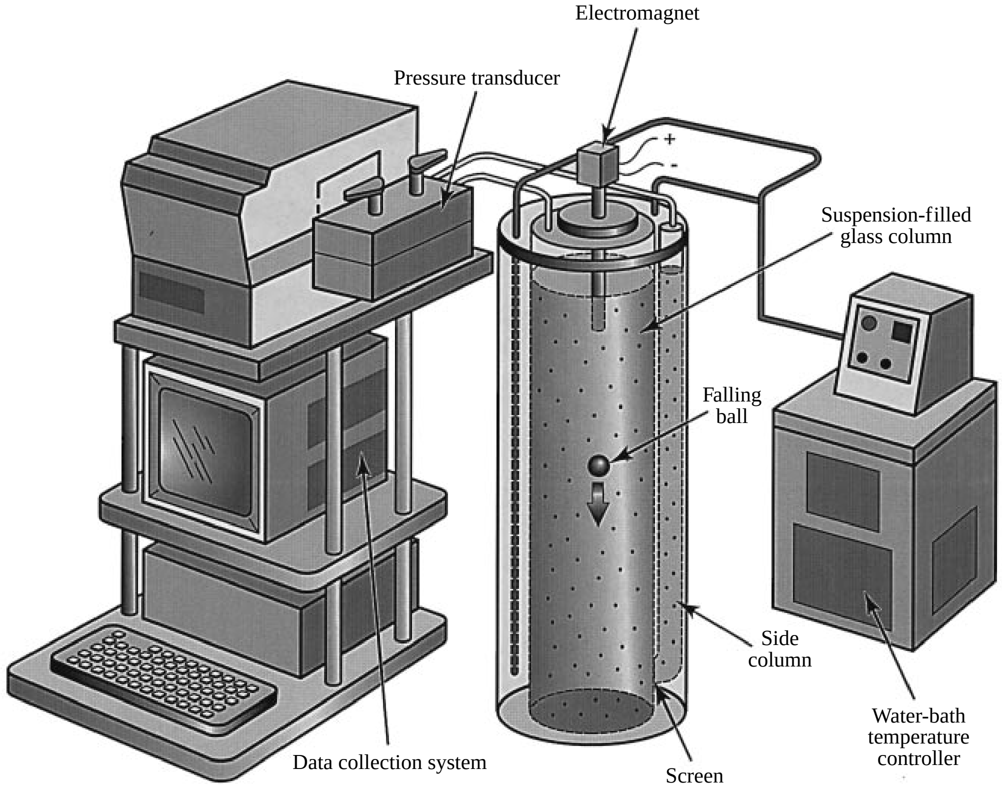

FIGURE 1. Sketch of experimental apparatus.

significantly lower pressure differences with the smallest balls, to test the accuracy of the present apparatus we first performed experiments on the pure suspending liquid (for which the theoretical results had already been experimentally verified by Pliskin & Brenner (1963)).

Our approach to measuring the differential pressure using two communicating cylinders was based on the earlier experiments of Pliskin & Brenner (1963). However, as discussed by these authors, this design has the disadvantage of rendering the measurements susceptible to small temperature variations. To mitigate any such effects, we placed both cylinders in a large water bath through which coolant water was recirculated by a Brinkmann RC20 temperature controller (Brinkmann Instruments, Westbury, NY). Bath temperature was maintained constant to within 0.01 °C. However, stirring the suspension was observed to create a small temperature rise in the suspension. Concern also existed that introduction of the falling ball could also change the temperature, albeit slightly. Therefore, after introduction of the falling ball – but before releasing it – the suspension was allowed to stabilize (usually between 30 and 60 minutes) until the differential pressure between the two cylinders no longer changed.

The balls dropped were magnetic, either steel or tungsten carbide, with nominal densities of 7.8 and 15 g cm^{-3}, respectively. Each ball was carefully weighed to determine its mass m to ± 0.0002 g and its diameter $2a$ to ± 0.0001 cm. The local acceleration due to gravity g was determined in a nearby building with a LaCosste & Romberg Model D gravity meter to within an accuracy better than 1 milligal (Gannett Fleming West, Inc. 1995). Finally, the density ρ of the liquid was measured with a Parr Calculating Digital Density Meter DMA45 (Anton Paar, Graz, Austria) to be

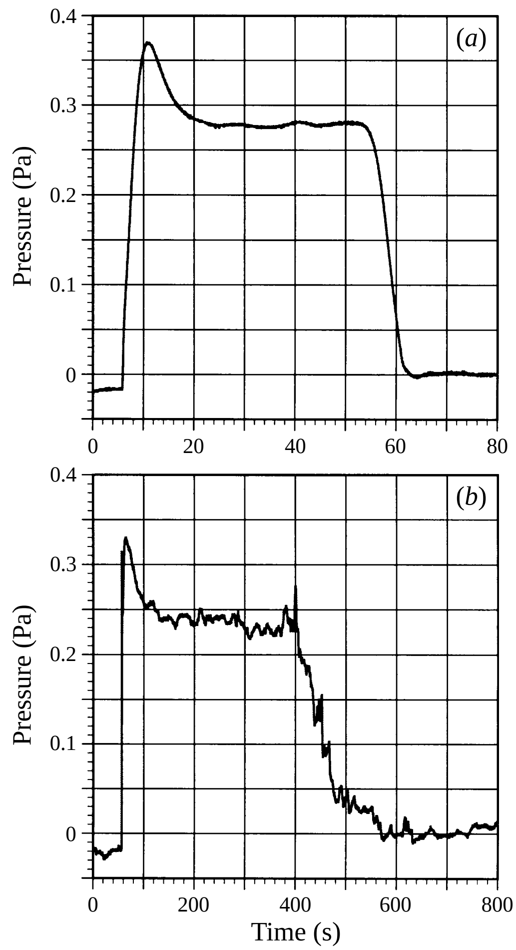

FIGURE 2. Example pressure trace for a 0.159 cm radius tungsten carbide ball falling in (*a*) the (single-phase) suspending liquid, (*b*) a suspension of 0.318 cm radius neutrally buoyant spheres (volume fraction = 0.50).

1.1812 ± 0.0003 at the operating temperature. Therefore, the product $(m-(4/3)\pi a^3 \rho) g = D$ was known to a high degree of accuracy. Seven ball sizes were used, with nominal radii ranging from 0.1588 to 0.6350 cm in increments of 0.0794 cm ($0.02 < a/R_0 < 0.08$). With use of an electromagnet connected to the cap that sealed the top of the larger cylinder, the balls were held partially submerged in the liquid. The electromagnet was situated in the centre of the cylinder for the initial experiments. Later experiments were performed in the Newtonian oil solely for the purpose of verifying (3). During such studies, the electromagnet was held at a known distance b from the axis ranging from $b/R_0 = 0.11$ to 0.75.

After the initial differential pressure appeared to stabilize, the electromagnet was turned off, the ball released, and the voltage signal from the differential pressure transducer continuously recorded (by an IBM PC equipped with analog-to-digital data acquisition capabilities) until the settling ball reached the cylinder bottom. The data were then transferred to a more powerful computer for analysis.

Typical pressure traces (figures 2a and 2b) exhibited a sharp rise as the ball was released, an overshoot, a steady pressure reading as the ball traversed most of the cylinder, and then a somewhat slower decline as the ball settled on the bottom. The ball was visually monitored during an experiment, and the time interval noted during which the ball was present in the centre third of the cylinder. Data recorded during this interval were used to obtain the average maximum pressure. This average was observed to be relatively insensitive to the number of data points taken as well as to the exact beginning and ending point chosen; nevertheless, we tried to be consistent across all experiments. The average static pressure registered shortly after the ball reached the bottom and ceased moving was used as the 'zero' reading, and subtracted from the average maximum pressure in order to obtain the differential pressure of interest. The pressure at the end of the experiment rather than at the beginning was taken as the baseline due to the fact the ball was usually not fully submerged at the beginning of a test (in order to facilitate its release from the magnet).

3. Results

At the outset, the apparatus was tested by measuring the pressure differences attending the settling balls falling in the homogeneous suspending fluid from which suspended particles were absent. Figure 2(*a*) is an example of the pressure trace for a 0.159 cm radius tungsten carbide ball. Ten balls were dropped for each of the seven nominal sizes, the average C_P value for each ball size being plotted in figure 3. At these a/R_0 ratios the maximum value of the wall correction term is less than 0.005. (In other words, C_P for the largest ball is predicted to be 1.99 rather than 2.00, according to (3).) The 95% confidence limits (Namely $tS/n^{1/2}$, where S is the standard deviation of the measurements, $n = 10$ the number of measurements, and $t = 2.626$ from standard statistical tables) are included for each point. The smallest ball gives rise to the largest uncertainty, as expected when measuring the extremely small pressures differences involved (approximately 0.3 Pa). In addition, the smallest balls took significantly longer to fall; and the longer the experiment, the more likely that small temperature variations could lead to spurious pressure measurements. Nevertheless, the average of the data points for each nominal ball size is close to the expected value. Moroever, the average of all seven values of C_P is 2.018, within 1% of the theoretical value.

Figure 2(*b*) shows a typical pressure trace for a 0.159 cm radius tungsten carbide ball falling through the $\phi = 0.50$ suspension. The pressure response for the suspensions displayed more short-time fluctuations than observed in the homogeneous fluid, presumably due to the interactions of the falling ball with the suspended spheres. This can be seen by comparing figures 2(*a*) and 2(*b*).

Pressure-drop-coefficient data taken in the suspensions with solids volume fractions of 0.30 and 0.50 are shown in figures 4(*a*) and 4(*b*), respectively. The concomitant increase in apparent viscosity caused the settling time for a ball to increase as the concentration of suspended particles increased. Compared with the suspending liquid alone, this resulted in the measurements taking over 10 times longer in the most concentrated suspension. And, as noted above, these relatively long experiments were particularly susceptible to temperature variations. Therefore, it is not surprising that the confidence limits increase with increasing solids concentration.

For the $\phi = 0.30$ suspension the average of all seven C_P values was 2.001, very similar to the results obtained for the homogeneous Newtonian oil. In contrast, however, the average C_P value obtained for the $\phi = 0.50$ suspension was 1.803. There appears to be no discernible effect on C_P of the relative ball/suspended-sphere size, a/a_s; thus, a ball half the size of the suspended particles yielded the same pressure drop force/drag ratio as one with a diameter twice that of the suspended particles.

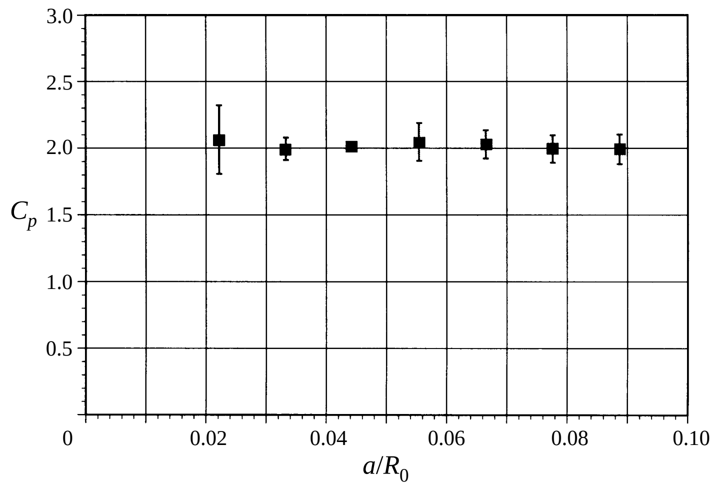

FIGURE 3. C_P values (equation (2)) measured in the (single-phase) suspending liquid for various falling ball sizes relative to the containing cylinder.

During the suspension experiments, the falling ball was sometimes observed to wander from its axial position within the cylinder. As evidenced by (4), the expected wall effects for a Newtonian liquid vary with the distance b. We have no reason to doubt that they would also do so were the fluid a suspension. Off-axis wandering of the ball can therefore have two effects. First, because each ball traces a different path through the cylinder, each could experience, on average, a different wall effect. This phenomenon could contribute to the spread in values of the pressure drop measurements and, hence, to the magnitude of the experimental uncertainty. Furthermore, as can be seen from (4), a non-axially-positioned ball will, according to the theory for a homogeneous fluid, always manifest a smaller C_P ratio than one situated on the axis. As such, the average C_P value measured in suspensions where the ball spent an appreciable proportion of time away from the cylinder axis could be significantly less than for a homogeneous liquid where the ball remained permanently along the axis.

In order to estimate the effect of the ball's migration, we first verified (4) for a homogeneous Newtonian liquid. Several tungsten carbide balls with a nominal radius of 0.3175 cm were dropped at seven non-axial locations in the cylinder filled with the Newtonian suspending liquid. Steel balls, each of a nominal radius of 0.635 cm, were also dropped at five of those locations. The measured C_P values are shown in figure 5 and compared with the values predicted from (4). Experiment and theory clearly agree quite closely.

Balls falling through a suspension will generally wander too far and erratically to verify (4). However, we have precedent to believe that the effects would be close to those in a single-phase Newtonian liquid. The effects of the cylinder walls on the velocity of a falling ball have been studied for various relative sizes of falling balls, suspended particles, and containing cylinders, and have been shown to be the same in suspensions of spheres and rods as those in a Newtonian liquid over an appreciable range of these parameters (Mondy *et al.* 1986; Milliken *et al.* 1989*a*, *b*).

Estimates of the effects of the falling ball's possible lateral migration were effected by combining earlier measurements of the horizontal and vertical dispersivities of falling balls (Abbott 1993) with a computational algorithm so as to model the pressure

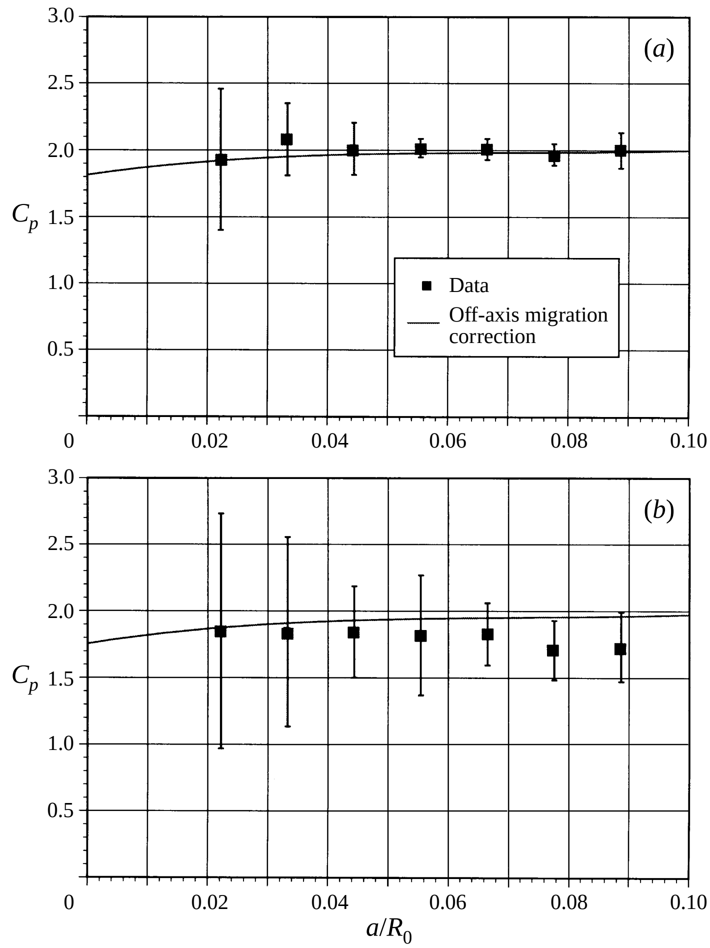

FIGURE 4. C_P values (equation (2)) measured in (*a*) the $\phi = 0.30$ suspension and (*b*) the $\phi = 0.50$ suspension for various falling ball sizes relative to the containing cylinder. The solid line represents the C_P values predicted by an algorithm that accounts for the migration of the falling ball away from the cylinder axis.

drop. These dispersivities were measured under the same experimental conditions as in the present laboratory experiments, except that the suspended spheres were half the size. Two methods were used to estimate the mean displacement off the centreline, as the falling ball passed the column's midpoint. The first method used the equation

$$b = (D_h\, l/v)^{1/2}, \tag{5}$$

where D_h is the horizontal dispersivity correctly scaled for the relative sizes of the suspended spheres and falling ball (Abbott 1993), l is the length of the cylinder, and v is the average settling velocity. This method neglects any interaction between the vertical and horizontal dispersivities. The second method used an algorithm that calculated the ball's path as a random walk, assuming the ball's motion to be governed by Gaussian distributions. The mean vertical displacement was assumed to be the average settling velocity of the falling ball multiplied by a constant time step. The mean horizontal displacement was assumed to be zero. Finally, the variances of the Gaussian distributions were set to be twice the respective (experimentally derived) dispersivities

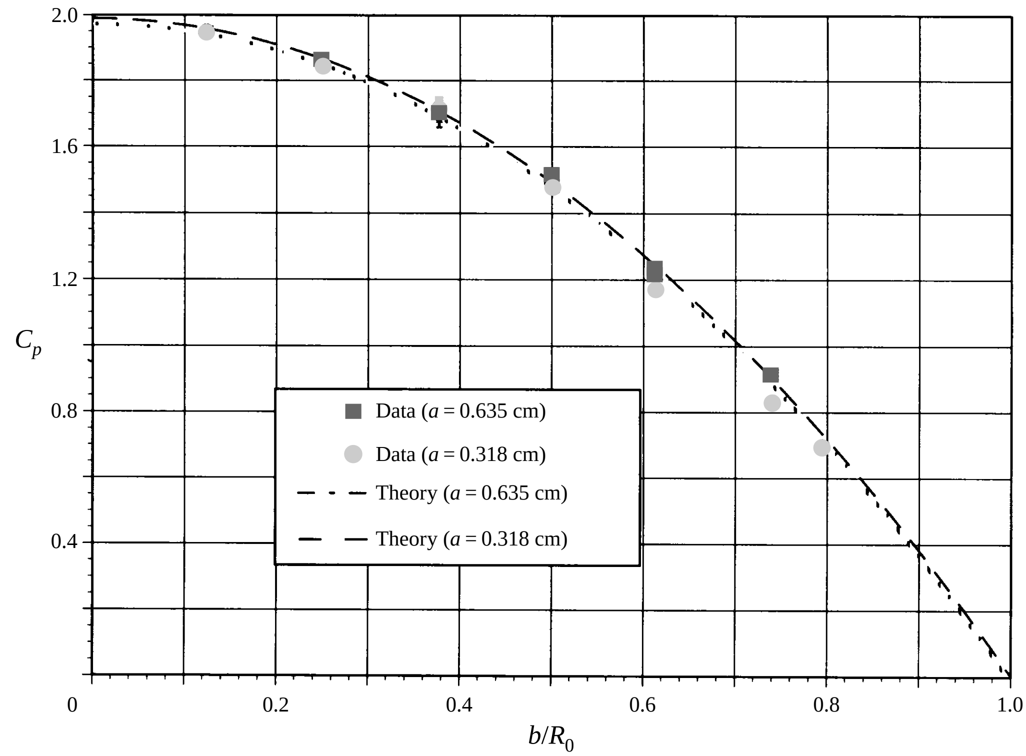

FIGURE 5. Measured C_p values (equation (2)) for balls falling non-concentrically in the (single-phase) suspending liquid a distance b from the containing cylinder (radius R_0) axis. The dashed lines represent the C_p values predicted by equation (4).

multiplied by the constant time step. Each ball was released at the centreline of the column and allowed to settle until it had traversed more than half the column height. Results obtained by repeating this experiment 20 times and averaging the mean off-centre distance at half the column height were, within error limits, indistinguishable from those obtained by the simpler method embodied in (5). Using the value of b obtained from the latter equation, (4) was used to determine the pressure drop appropriate to a homogeneous Newtonian fluid.

Mean values of C_P corrected for the falling balls' migration as predicted by our algorithm (as a function of the ball size) are shown as the lines in figures 4(a) and 4(b). Although migration of the ball away from the cylinder axis does lower the predicted value of C_P, and although the experimental uncertainties in the measured pressure drop are in most cases large enough to accommodate the prediction, the migration of the ball may not fully account for the lower value of C_P seen in the $\phi = 0.50$ suspension. In fact, at least one data point in figure 4(b) is too low to match the predicted C_P value.

4. Discussion and conclusions

The pressure drop across a ball falling in a moderately concentrated suspension ($\phi = 0.30$) matched that expected for a homogeneous Newtonian fluid. The ratio C_p of the measured pressure drop force (the pressure drop ΔP multiplied by the cross-sectional area A of the containing cylinder) to the drag force D on the ball was found to be close to the theoretical value of 2.0. The size of the ball relative to that of the suspended particles had little effect, if any, over the range of parameters ($0.5 < a/a_s < 2$) encountered in our study for both this suspension and the more concentrated one. It is interesting to note that in earlier studies the drag on a ball falling through a suspension of like spheres could be very sensitive to the relative ball/suspended-sphere size, a/a_s, decreasing dramatically as a/a_s decreased below 0.8 at $\phi = 0.50$ (Milliken *et al.* 1989*b*). In contrast, under similar conditions the pressure drop is seemingly independent of a/a_s (as well as of the apparent viscosity).

However, the uncertainties in the pressure measurements in the suspensions were larger than those observed in the comparable homogeneous particle-free suspending Newtonian oil case. A combination of effects was likely to cause these larger uncertainties. First, the measured pressure difference fluctuates more extensively during an experiment performed with a suspension than it does with the suspending liquid alone. These pressure fluctuations appear to originate from the discrete nature of the suspension, which also causes the ball to follow a seemingly erratic path with frequent excursions in instantaneous velocity that are obvious to the observer. The ball may also wander a considerable distance from the cylinder centre. (The smallest balls migrated up to 40% of the distance to the cylinder wall in the highest volume fraction suspension.) Because each ball followed a different path, the average wall effects varied from one experiment to the next, increasing the spread in the measured pressure drop. Finally, because the apparent viscosities are much higher in the suspensions than in the homogeneous fluid, the ball takes a much longer time to traverse the apparatus. This increases the probability of errors arising from temperature fluctuations.

The mean pressure drop was somewhat lower in the more concentrated of the two suspensions; however, the uncertainties in the measurements are so large that the result from only one ball size ($a = 0.159$ cm) can be statistically proven to differ from that expected in a Newtonian liquid. Nevertheless, the mean C_P value for each ball size was consistently lower than 2.0, hovering around 1.8. This can only be partially explained by off-axis migration of the ball. As part of this study we verified the accuracy of the predictions (Brenner & Happel 1958; Brenner 1962; Bungay & Brenner 1973) of the effects on C_P of non-axial settling ($b/R_0 \neq 0$) in homogeneous fluids.

Concentrated suspensions of spheres dispersed in Newtonian liquids may exhibit apparent shear thinning in a conventional rheometer (Krieger 1972; Gadala-Maria 1979). As such, it could be argued that the lower C_P values observed were consistent with the Ribeiro *et al.* (1994) conjecture that this ratio is lower for shear-thinning than Newtonian liquids. However, apparent shear thinning was detected in none of the previous experiments wherein the suspension's apparent viscosity was measured via falling-ball rheometry under conditions virtually identical to those in the present experiments (Mondy *et al.* 1986; Abbott 1993). These facts lead us to speculate that the lower-than-expected pressure drops observed in the suspensions tested (and, perhaps too, in Ribeiro *et al.*'s (1994) most concentrated polyacrylamide solution) were caused by suspension-scale 'slip' at the cylinder wall or from a blunting of the non-Newtonian velocity profile arising from the presence of suspended particles.

By measuring the velocities of the suspended particles in tube flow, Karnis, Goldsmith & Mason (1966) observed slip in suspensions. Similar observations have been reported for Couette flow between rotating concentric cylinders (Jana, Kapoor & Acrivos 1995), in addition to having been inferred in a host of other studies (for example, Vand 1948*a*,*b*; Higginbotham, Oliver & Ward 1958; Seshadri & Sutera 1970; Yoshimura & Prud'homme 1988; Yilmazer & Kalyon 1989; Boersma *et al.* 1991). Apparent slip at the wall in suspensions is believed to occur because the finite size of the suspended particles leads to the existence of a thin layer near the wall characterized by a smaller solids concentration than in the bulk and, hence, possessing a lower apparent viscosity than the average. (This phenomenon is sometimes called the Vaud effect.) Recent studies have shown that the particles in concentrated suspensions can migrate away from walls in channel or tube flow, even at very low Reynolds numbers (Leighton & Acrivos 1987; Phillips *et al.* 1992; Koh, Hookham & Leal 1994; Nott & Bray 1995; Hampton 1996). Such migration can intensify the apparent slip effect, but can also complicate the detection of effect due solely to steric exclusion.

Apparent slip accompanying laminar flow has also been documented in the polymer solution and melt literature, and has been speculated to be caused by depletion of polymer molecules in the wall region (for example, Mooney 1931; Kozicki *et al.* 1970; Carreau, Bui & Leroux 1979; Cohen & Metzner 1985). Cohen & Metzner (1985) saw slip effects even in large channels (channel size/macromolecular size > 100) with solutions of polyacrylamide.

The pressure drop across a ball falling through a liquid bounded by a cylinder is crucially dependent on the slip/stick boundary condition at the cylinder walls (Brenner 1962). For perfect slip the C_P ratio is 1.0, whereas it is 2.0 with perfect stick. In effect, with perfect slip the wall experiences no shearing force. (In contrast the drag on the falling ball is modified by the presence of the no-slip boundary condition at the cylinder wall only by the Faxén wall correction (Faxén 1923; Bohlin 1960), which in the present geometry is reasonably small.) Therefore, because of sensitivity to slip, pressure drop measurements for a ball falling in a quiescent suspension (or even in a polymer solution) may provide an attractive means of quantifying the effects of steric exclusion of the particles (or macro-molecules) near the walls, without the added complexities arising from the formation of shear-induced 'slip' layers accompanying net flow of the material.

An alternative explanation of the lower-than-expected pressure drop may be the blunting of the otherwise Poiseuillian parabolic velocity profile, observed to occur during the laminar flow of concentrated suspensions in tubes (Karnis *et al.* 1966). For circumstances other than the homogeneous Newtonian-fluid circular-tube case, the generic expression for the pressure-drop coefficient defined in equation (2) is (Brenner 1962)

$$C_P = v_0^0/V_m \tag{6}$$

where v_0^0 is the approach velocity of the suspension to the point (situated at the axial position b) where the centre of the falling ball is located, when the neutrally buoyant suspension flows through the duct with mean velocity V_m. It is this generic relation that leads to equation (4) for the Poiseuille flow case (at least in the absence of wall effects, $a/R_0 = 0$) as well as to the power-law flow expression, $C_P = (s+3)/(s+1)$, of Ribeiro *et al.* (1994) for the concentric sphere location case, $b = 0$, cited earlier. The blunting of the Poiseuille velocity profile revealed by the experiments of Karnis *et al.* (1966) on the flow of concentrated suspensions corresponds to an axially situated sphere value of $v_0^0/V_m < 2.0$. This inequality is consistent with our experimentally observed value of $C_P \approx 1.8$ for the 50% suspension case, although the experimental values reported by Karnis *et al.* (1966) for the ratio $u'(0)/V_m$ at their highest concentration of 34 volume % (with $u'(0)$ the centreline velocity of the plug flow) would suggest a significantly smaller value than 1.8, namely 1.5–1.6 at their 34 volume % concentration. If the latter interpretation is correct, then experimental measurements of $C_P = \Delta PA/D$ in quiescent suspensions at various fractionally eccentric sphere positions b/R_0 may provide an alternative scheme for establishing the velocity profile in concentrated suspensions of neutrally buoyant particles.

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